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By- Vincent, William S.

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This paper attempts to show that adaptation of mathematics to the input-output model of the school can provide powerful assistance in the measurement and analysis of school quality and its determinants. The mathematical relationship described here relates an educational model to the field of electronics. More specifically, the amplifier, a device which increases the magnitude of an input, in discussed as being analogous to the IQ-academic achievement relationship. (hw)

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The School As An Amplifier

William S. Vincent

In earlier issues of the Bulletin[1] an input-output model of a school system was presented as a basis for discussing the problem of quality measurement. The school was classified among those operations which have both an instrumental input and a purposive input. The two inputs were distinguished as follows: With institutions or operations of a certain class the principal reason for the existence of the institution is to receive a particular input and to process it. Such an input is a *purposive* input. If there were no purposive input (in the case of the school, pupils) there would be no need of the institution (the school). All operations, even those without purposive inputs, have an *instrumental* input. The instrumental input consists of matériel, personnel, financing, management, and organization. Criteria of quality, it is proposed, are of two kinds: one obtainable as a measure of output; the other an evaluation of internal process. It can be shown that quality criteria based exclusively upon output are not completely valid for institutions with a purposive input. In the earlier discussion a diagram was presented to show not only input, process, and output, but also certain other quantities critical to the operation such as input generators, environmental influences, and feedback loops.[2]

The term "model" as employed today to represent the conceptualization of an operation does not necessarily refer to a diagram or schema, although some pictorial program is usually inferred. In particular the term implies translation of the operation into mathematical terms. Thus a model belongs to a class of formulae, and makes it possible to relate the parts of the operation to each other, to fit the operation into the matrix of its environment, to do this quantitatively, and to explore mathematically the effect that variation of internal and external quantities has upon some quality criterion.

Missing from the earlier articles referred to was any attempt to show that there are mathematical analogies which can be applied in this case. The usual procedure is to seek an analogy rather than invent a new mathematics. For, it would appear, pure mathematics is pursued without regard to possible physical applications, and there is much more of it than has ever been applied. Furthermore, so great is the generalizing power of mathematics that an application to a first purpose frequently suggests application to a second purpose when the conceptual patterns of the first and second purposes are analogous.

The intent of the following discussion is to show that the model as presented possesses conceptual analogies to another version which has had extensive mathematical treatment. It is suggested that adaptation of this mathematics to the input-ouput model of the school can provide powerful assistance in the measurement and analysis of school quality and its determiners.

A Mathematical Model

The mathematical relationships that describe this model derive from the field of electronics. For the school is a kind of amplifier—as are all purposive input opera-

[1] *IAR Research Bulletin*, "Measuring School Quality: Output and Process," Vol. 4, No. 3, May 1964; "Inputs and Criteria," Vol. 6, No. 1, November 1965.

[2] *Ibid.* Vol. 6, No. 1.

tions. An amplifier is a device which can increase the magnitude of an input. In electronics this input is termed the "signal". It is an alternating or modulated voltage whose magnitude is increased within the amplification circuit by means of current which flows from a power supply. Thus there are in effect two types of input: the input signal and the power supply.

The input signal is a purposive input. For it is the sole purpose of the amplifier to take in the weak signal and to amplify it so that it may be used to drive a loudspeaker or accomplish some other work. If the signal were not incapable of accomplishing the desired work to begin with, there would be no need of an amplifier. The power supply is an instrumental input. Its sole function is to supply whatever energy may be needed to transform the weak input signal into an amplified output signal.

Thus an amplifier is a *purposive input* device. It may serve to provide its analogue, the school, with a mathematical model which can be used to clarify some of the forces which determine the capacity of a school to take in a "weak signal" and transform it into an "amplified output" capable of accomplishing desired work not otherwise possible to it. Further examination of the analogy reveals that the electronic version of the model is

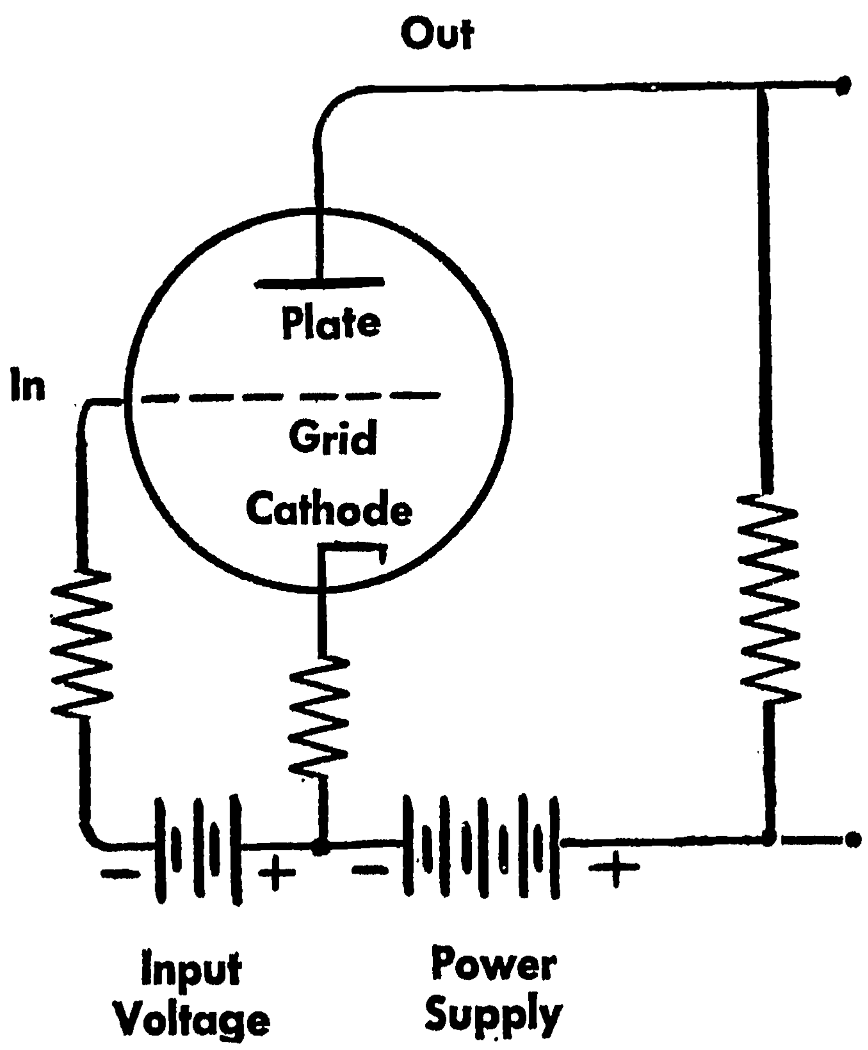

Figure 1: Circuit Design of a Single Tube Amplifier

relatively simple compared to the school version, which is quite complex. Nevertheless major factors in both operations provide analogies of amazing similarity. Graphically, the similarity may be perceived by comparing the earlier input-output diagram with Figure 1, which is a simple circuit design of a single tube amplifier.

The amount that an amplifier can increase the magnitude of the input signal is called the *gain*. The gain is dependent upon internal operating conditions as these are to greater or less degree capable of drawing current from the power supply, or instrumental input, to increase the magnitude of the signal, or purposive input. Gain is defined as the ratio of signal output (o) to signal input (s):

$$(1) \quad G_{so} = \frac{X_o}{X_s} = \frac{\mu R_o}{r_p + R_o}$$

for which a simple block diagram* is:

$$X_s \longrightarrow \boxed{G_{so}} \longrightarrow X_o$$

The symbol X in this and subsequent formulas and diagrams stands for quantities measured at different points in the circuit and the subscripts denote the points. R is the conventional sign for resistance, and μ (or mu) and r_p (which stand for amplification factor and plate resistance respectively) define certain critical operating conditions.

Formula (1) above provides a principal mathematical definition of an amplifier and contains equivalent expressions, one of which is an output measure and the other a process measure. The ratio of output to input $\frac{X_o}{X_s}$ is obviously an output measure. Amplification factor (μ) is a process measure. It describes a condition within the amplifier, or more specifically, relationships among the operating parts of individually functional parts of the amplifier. In order to evaluate the effectiveness of an amplifier as a function of its amplification factor one would have to know many things critical to its internal operation: the number of amplifying units (i.e. the number of tubes or transistors), the amplification factor of each unit (which in a tube, for example, is a function of the distance between grid and cathode compared to the distance between anode and cathode), the manner in which the units are coupled (i.e. whether direct coupled, impedance coupled, capacitor coupled, or

*Adapted, as are other block diagrams, from Allen, Charles L. and Atwood, Kenneth W., *Electronic Engineering*, John Wiley, New York, 1962.

transformer coupled), the ratio of internal resistances and voltages.

Effectiveness of an amplifier refers to its capacity to perform work of various kinds. Irrespective of what its gain may be (its output measure), its internal operating processes are more critical to its particular application. Whether the amplifier is to be employed in a circuit to reproduce music, detect a radio frequency, create an oscillating signal, control a photo electric unit, or perform any one of the hundreds of other uses to which amplifiers are put, more important than gain is the matching of the conditions of internal operation to the particular application. Internal conditions relate not only to amplification factor, but method of tube biasing, and various types of circuit controls over such factors as those designated by the terms *resistance, capacitance,* and *inductance*. In other words, the matter of evaluating an amplifier by measuring its internal conditions of operation is a complex problem requiring considerable skill and knowledge of amplifier internals and no ambiguity regarding the purpose the amplifier is to serve.

A sensitive ear may perhaps detect the similarity between these limitations and those discussed previously[3] in connection with the complexity of employing process measures to evaluate a school, and the relation between process and output measures as criteria. The principal criterion of the school, viewing its output, is its measure of gain. Yet a measure of gain alone, without regard to application (i.e., objectives of the school) would yield little for purposes of evaluation. The advantage to the electronic engineer is that all of the quantities with which he deals can be measured, and they all have been shown to correspond to mathematical functions. But if it can similarly be shown that the analogue is applicable to the school, the mathematical models that guide the engineer in his investigations can be adopted or adapted to the investigation of the internals and the "circuit controls" of this other kind of amplifier whose purpose is similarly to augment the purposive input.

Further Similarities Between Schools and Amplifiers

The capacity of electronic circuitry to yield to mathematical analysis is enhanced by the convertibility of electrical measures. It might be said that the task of the electronic mathematician is made easy by Ohm's Law and Watt's Law. The base quantities of electronic circuitry—*voltage, current, resistance, power*—may be interchanged within the restrictions of these two formulas. Inherent to the mathematical validity of the electronic model, therefore, are units of measure that have fixed means of convertibility.

Education possesses analogous measures. The interchangeability of quantities represented by

$$(2) \qquad IQ = \frac{MA}{CA}$$

is fully as important to the model of the school that we have under discussion as Ohm's Law or Watt's Law to the electronic model. Also,

$$(3) \qquad Y = \beta_0 + \beta_1 (X - M_x) + \beta_2 (X - M_x)^2 + \beta_3 (X - M_x)^3 \ldots,$$

which represents an output prediction (in terms of an achievement test measure) as controlled by a measured relation between the achievement test and an acceptable IQ test, is a necessity to the application of mathematics to the school model.

What is missing at the moment is a formula defining the interchangeability of purposive input-output measures with instrumental input and process measures. In other words, we do not have at the moment established means for converting "volts" to "current" and "resistance". We do not know how to convert dollars to units of staff or how to predict units of effort from units of wealth for given financing plans. However, the results of many studies hint at what the relations between various inputs and other measures may be. The probability is that we do not have conversion formulas because we have not looked for them.

Formula (3) above, it will be noted, is a prediction based upon a non-linear regression. Data collected by Atkins[4] show that the shape of the regression of an achievement measure upon an IQ measure is nonlinear. In the upper range of IQ the rate of increase of achievement decreases with increasing IQ. Though his data do not clearly show it, the presumption is strong that the opposite effect occurs in the lower range of IQ —i.e., that the rate of decrease of achievement decreases with decreasing IQ (see Figure 2). This phenomenon is of course a function of the measuring instrument but is probably inherent in all such devices.

Thus the relation between IQ and achievement is distorted at the extremes of the IQ range and linearity appears only in the mid range. This circumstance can, of course, greatly influence the reliability of an output

[3] *IAR Research Bulletin*. Vol. 4, No. 3.

[4] Thurston Atkins, "Elementary Achievement Test Survey," Special Report, Associated Public School Systems, 1962.

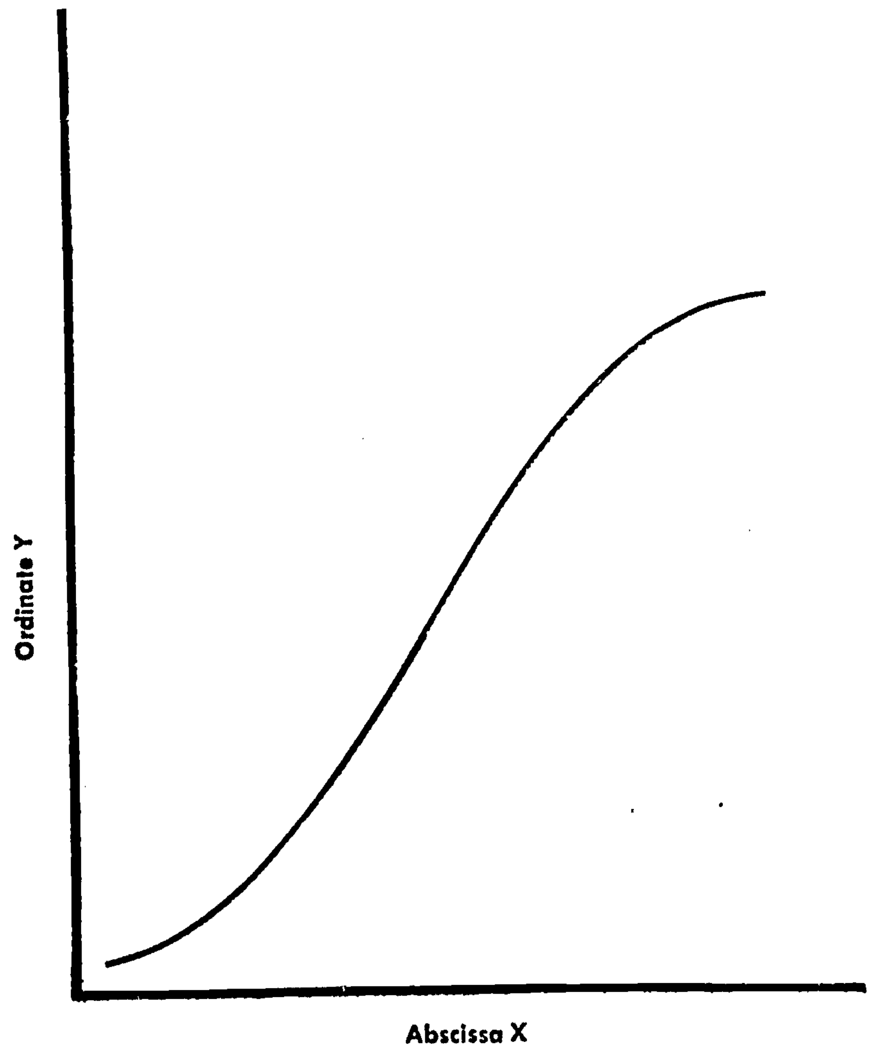

Figure 2: Distortion at the Extremes of the Range
In the electronic version, Abscissa X is the signal input voltage, Ordinate Y is output current; in the school version, Abscissa X is IQ, Ordinate Y is achievement measurement.

measure obtained on a school whose distribution of pupil populaton by IQ is abnormal.

The principal interest of this phenomenon, however, is that it parallels the relation between input voltage and plate current in a vacuum tube. What happens in a vacuum tube when it amplifies an input signal is roughly as follows:

The more positive the input signal voltage becomes, the greater the current in the output circuit. But as the input becomes very positive, its ability to increase output current decreases. A voltage increase in the midrange results in a current increase of a magnitude normally to be expected as a result of the tube's amplification factor. But at the upper range of the positive-negative cycle the resulting change in the amplified current is greatly reduced. In fact there gradually comes a point (saturation) at which no amount of increase in the positive polarity of the input can produce amplification in the output. Similarly, at the negative extreme, as the input becomes more and more negative, its ability to decrease output current decreases. There gradually comes a point (cutoff) where no amount of increase in negative polarity will have any effect upon output current.

Thus a tube has to be "biased". (The same phenomenon and a similar treatment also apply to the tube's substitute, the transistor). The grid of a vacuum tube, on which the input signal enters, has to be made less negative in potential than the cathode, the element from which current flows across the tube interior to the plate. The reason for this is because electrons, whose flow from negative to positive produces current, are negative in potential. If the grid is positive with respect to the plate, the electrons will collect at the grid and not move on to the plate. Thus there will be no current. If the grid is negative with respect to the cathode it will repel the negative electrons, not permitting them to cross over to the plate and there will be no current either. The only way current may flow is to make the grid more negative than the plate and less negative than the cathode. Somewhere between the extremes of cathode and plate the grid functions as an electron gate. The incoming signal, whose potential varies, also varies the potential of the grid. When the signal is positive the grid is less negative, permitting more electrons to flow; but if it becomes too little negative, a large change in the signal will result in too small a change in electron flow and saturation, a form of distortion, will result. When the signal is negative the grid is more negative than before, permitting more electrons to flow than before; but if it becomes too negative, electrons will be repelled back to the cathode and no current will flow; thus "cutoff" of the most negative portion of the input signal creates another form of distortion. (See also Figure 2.)

Thus the relation between input signal and output current is distorted at the extremes of the voltage range and linearity appears only in the mid range. Engineers optimize the operating range of the tube by establishing its operating point, or "biasing" it, to the mid range. Their circuit designs do not permit tubes to operate outside this range. Because of a peculiarity imposed by a natural exigency (the creation of current by negative electron flow) the tube has to be biased and operated as "more negative" and "less negative" rather than as negative and positive, the way the input signal comes in. Similarly because of a peculiarity imposed by a natural exigency (the relation between measured IQ and measured achievement) a measurement "bias" or control of the IQ factor has to be imposed on the purposive input of the school in order to avoid a distorted reading of the output. Not only is such a form of "biasing" required but the analogue of optimizing the operating range needs to be applied. Correcting for IQ requires recognition of the fact that the relation between achievement and IQ is non-linear at the extremes. Since conventional correlations between achievement and IQ, such as those supplied by test makers, assume for the whole IQ range a linearity that exists only in the mid range, adjustments must be made in output measures of that part of pupil input appearing at the extremes of the IQ range.

The input signal is said to *control* the output. Nothing flows at output except as augmentation of input. In the electronic version the purposive input is the determining factor in output. It can be seen in the modern educational view that this concept is also valid to the school as an amplifier. Purposive input controls output in the sense that instrumental input and operating conditions must meet all the requirements of the pupils. It is they who are the determining factor in the modern view, not the teachers, not the administration, not economic conditions; the achievement of high gain is permissible only when the instrumental inputs are fully sufficient to the needs of the purposive input.

The Feedback Loop as a Control Function

The performance of an amplifier may be altered by the use of *feedback*—that is, by adding all or part of the output signal to the input signal. If that part of the output returned to input is opposite in polarity to the original input signal, the feedback is said to be negative. In this case it is corrective, being used to reduce the overall distortion normally contributed by the circuit components of the amplifier. Distortion occurs in the amplification process as the result of faulty handling of the input signal. Indeed it may be said that the phenomenon of distortion is inherent in all such devices. There is residual distortion of the input signal by various circuit components so as to change its character in some way other than to do what the amplifier is supposed to do—merely increase its magnitude. Distortion present in the output is fed back to the input, but opposite in polarity, thus cancelling the distortion.

By extension, "negative feedback" is a term applicable to any action which reduces deviation. "Positive feedback" increases deviation. In an automatic guidance system, for example, slight irregularities in course are corrected by negative feedback, tending to keep the vehicle on its prearranged course. Positive feedback would increase random deviations and force the vehicle onto an erratic or completely different course. Negative feedback exerts a centralizing tendency; positive feedback contributes to deviation. Negative feedback is "conservative;" positive feedback is "experimental." The engineer calls the former "degenerative," the latter "regenerative," and applies each to specific purposes.

It is probable that feedback is widespread throughout nature. A combination of negative (emphasizing species regularity) and positive feedback (emphasizing mutation) is possibly the controlling influence in evolution. It appears that negative feedback is the "guidance system" whereby errors are gradually eliminated in trial and error learning and provides the basis for unifying not only all theories of learning but also learning and maturation.[5] One way of distinguishing between "teaching" and "telling" is that inherent in the teaching situation is negative feedback whereby the teacher's message and method become modified by return information indicating how well the pupils are learning.

In a managed operation, such as a school system, whether at any given time feedback is negative or positive depends upon the strategy of management. This strategy can opt for improvement in the process and elimination of deviation (negative feedback) or it can encourage experimentation through the augmentation of random changes (positive feedback). In any case the feedback loop is present in any managed enterprise whereby information obtained at output is fed back to modify inputs. The "feedback network" is an integral part of policy formulation and indeed is the central component in the institutional guidance system. What occurs in this component represents perhaps the major function of management.

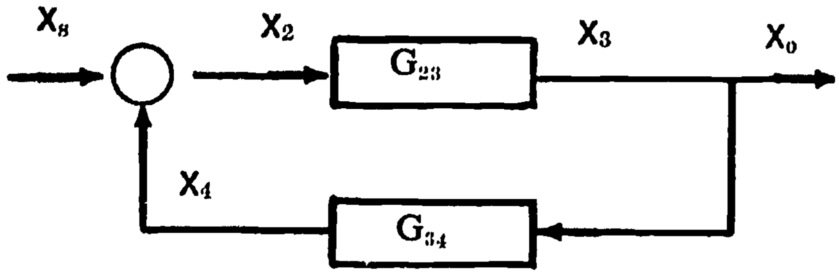

Figure 3: Amplifier with Feedback Loop

Employed in amplifier circuits negative feedback all but completely corrects for deficiencies in the internal process that cause mishandling of the input signal. A simple block diagram of an amplifier with feedback loop is shown in Figure 3. The gain of such a circuit is given in Formula 4:

$$(4) \quad G_{HO} = \frac{X_o}{X_s} = \frac{G_{23}}{1 + G_{23}G_{34}} = \frac{G}{1 + G\beta}$$

G_{34} (gain from X_3 to X_4) is often written β and

[5]See Jean-Pierre Changeux, "The Control of Biochemical Reactions," *The Scientific American*, Vol. 212, No. 4, April 1965, pp. 36ff. and Derek H. Fender, "Control Mechanisms of the Eye," *The Scientific American*, Vol. 211, No. 1, July 1964, pp. 24ff.

Table 1—Mathematics of a Feedback Amplifier Applied to Formula 4

$$
\begin{array}{ll}
(1) & X_3 = X_2G_{23} \\
(2) & X_4 = X_3G_{34} \\
(3) & X_2 = X_1 - X_4 \\
(4) & X_3 = (X_1 - X_4)G_{23} \\
(5) & \quad = (X_1 - X_3G_{34})G_{23} \\
(6) & X_3 = X_1G_{23} - X_3G_{23}G_{34} \\
(7) & X_3 + X_3G_{23}G_{34} = X_1G_{23} \\
(8) & X_3(1 + G_{23}G_{34}) = X_1G_{23} \\
(9) & G_{13} = \dfrac{X_3}{X_1} = \dfrac{G_{23}}{1 + G_{23}G_{34}}
\end{array}
$$

stands for *feedback factor.* Formula 4 states that the gain from signal to output for the circuit as a whole is equal to the gain from the input (X_2) to the output (X_3) for the amplification stage alone (G_{23}) divided by 1 plus this gain times the feedback factor (β). The formula results from simple algebra applied to the measured quantities of the diagram, and the steps are shown in Table 1. The constant in the formula (unity) derives, as may be seen at Step 6, from the fact that output, expressed partly in terms of itself, appears twice in the equation.

The expression (at Step 6) states that output is equal to input* as modified by the gain of the amplification stage minus the changes imposed by the feedback loop (G_{34}). Subsequent factoring (Step 8) leaves the numerical constant 1 in the expression. The feedback loop results in a reduction in gain since it appears in the denominator (Step 9).

The point of all this is that these same relations hold not only for the electronic device but for any enterprise with negative feedback. As the feedback loop reduces gain, so too the deliberation of management can reduce the ratio of output to input. That is to say, the more intricate the "administrative web," the less effective the enterprise, in exactly the same way that there is a limit to the amount of feedback possible in an amplifier without completely negating its amplifying function. Yet without feedback applied within its optimum limits the larger gain obtainable without it may be full of error. Thus, as in the electronic device, the design problem is to obtain just as much of feedback as will optimize both correction and gain. All testing and no teaching, as a simple example, would overwork the feedback loop and result in minimal gain.

Figure 4 shows a block diagram of a feedback amplifier with two input signals. Mathematically the part of the output (X_o) which results from one input (X_{s1} can be distinguished from the part which results from the other (X_{s2}).

In the first case

$$(5) \qquad G_{s1o1} = \frac{G_1G_2}{1 + G_1G_2\beta}$$

In the second

$$(6) \qquad G_{s2o2} = \frac{G_2}{1 + G_1G_2\beta}$$

Thus the model is applicable in the analysis of a school system to whatever need may exist for distinguishing among pupils (purposive input) of differing characteristics. There is some evidence to show that the capabilities of schools (their "gain characteristics") vary with respect to the type of pupil (as classified by mental, emotional, or socio-economic factors) with which they may have to deal. A school's operating procedures may deal quite successfully with pupils much above normal mentally, for example, but unsuccessfully with pupils of average mentality or pupils below normal.[6]

Discrepancies in the Analogy

While the analogy between the school and the electronic amplifier is a close one, the parallel is not perfect. The purposive input model as applied to a school system shows multiple inputs. However, there is the difference that the greatest variety occurs in the instrumental inputs—the financial inputs, staff inputs, administrative inputs, each class of input subject to a great variety of choices. The electronic model possesses an instrumental input (the power supply) which remains virtually unchanged irrespective of the exigencies of operating procedure or output.

There is a further salient difference. In the electronic version the feedback loop results in correction of the signal (the analogue of the purposive input of the school version). The change that feedback makes in the signal is actually a *predistortion* imposed at the input to

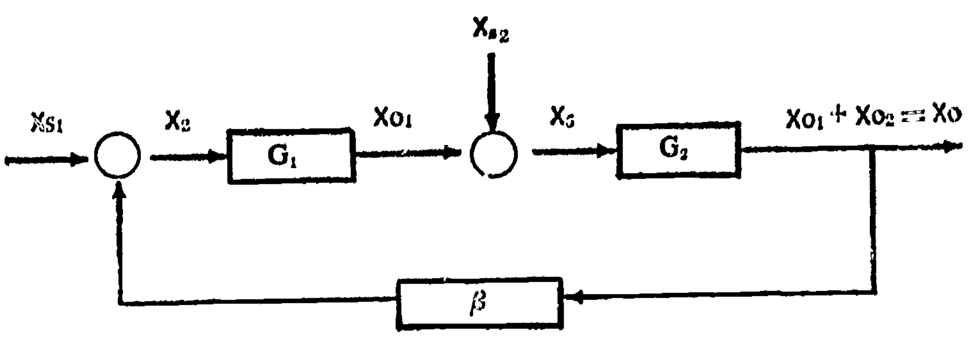

Figure 4: Feedback Amplifier with Two Input Signals

*Purposive input that is.

[6] Goodman, Samuel M., *The Assessment of School Quality*, Research Office, The State Education Department, Albany, New York, March 1959, p. 20.

compensate for distortion present in the amplification process. It goes without saying that no school does this. The saying that "the good school takes pupils from where they are" illustrates clearly enough that feedback does not result in changes in the purposive input. Corrections at input are in the *instrumental* inputs of the school. Compensations for purposive input occur in the process. This would be analogous to a feedback loop (in the electronic version) so designed as to effect changes in the whole circuit—create new amplification stages, insert resistors and capacitors as needed, in response to changing output. It would have to be admitted that no electronic amplifier can do this, at least at present. However, this kind of capability is exercised by management.

Thus it appears that the mathematics at hand for the analysis of the electronic amplifier requires some extension and development to deal effectively with certain of the peculiar complexities that distinguish the school as an amplifier. Nevertheless, the basic relationships are analogous. The capacity of the model to respond to mathematical investigation, once the significant quantities are measured, can be assumed. The advantage, then, is that the magnitude of the inputs, input generators, environmental conditioners, gain contributed by operating process, and feedback as an administrative function, can all be computed and related in the model.